Hong Kong Mathematics Olympiad (1989 – 90)
Heat Event (Group)
香港數學競賽 (1989 – 90)
初賽項目 (團體)

1. If $\frac{1}{a}+\frac{1}{b}=5$ and $\frac{1}{a^2}+\frac{1}{b^2}=13$, find the value of $\frac{1}{a^5}+\frac{1}{b^5}$.
若 $\frac{1}{a}+\frac{1}{b}=5$ 及 $\frac{1}{a^2}+\frac{1}{b^2}=13$，求 $\frac{1}{a^5}+\frac{1}{b^5}$ 的值。

2. There are N pupils in a class.
When they are divided into groups of 4, 1 pupil is left behind.
When they are divided into groups of 5, 3 pupils are left behind.
When they are divided into groups of 7, 3 pupils are left behind.
Find the least value of N.

某班有學生 N 人。
若將學生分為每 4 人一組，有 1 人餘下，
若將學生分為每 5 人一組，有 3 人餘下，
若將學生分為每 7 人一組，有 3 人餘下。
求 N 的最小值。

3. The coordinates of A, B, C and D are $(10, 1)$, $(1, 7)$, $(-2, 1)$ and $(1, 3)$ respectively. AB and CD meet at P. Find the value of $\frac{AP}{PB}$.
A、B、C 及 D 的座標依次是 $(10, 1)$、$(1, 7)$、$(-2, 1)$ 及 $(1, 3)$。AB 與 CD 相交於 P。求 $\frac{AP}{PB}$ 的值。

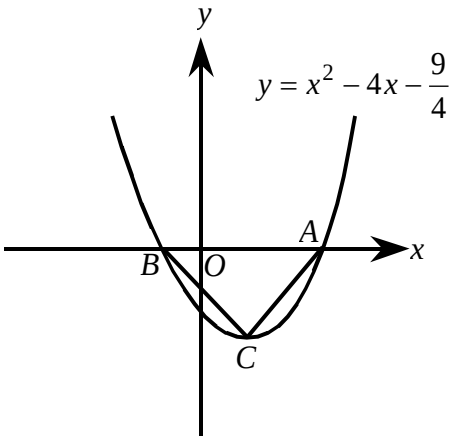
4. Find the remainder when $2^{1989}+1$ is divided by 3.
求 $2^{1989}+1$ 被 3 除所得的餘數。

5. Euler was born and died between 1700 A.D. and 1800 A.D. He was $n+9$ years old in n^3 A.D. and died at the age of 76. Find the year in which Euler died.
歐拉在 1700 A.D. 和 1800 A.D. 之間出生和去世。在 n^3 A.D. 時，他剛好 $n+9$ 歲，而他在 76 歲時去世。求歐拉去世的年份。

6. Let $N!$ denote the product of the first N natural numbers, i.e. $N!=1\times2\times3\times\cdots\times N$. If k is a positive integer such that $30!=2^k\times$ an odd integer, find k.
設 $N!$ 為首 N 個自然數的積，即 $N!=1\times2\times3\times\cdots\times N$。若 k 是正整數使得 $30!=2^k\times$ 一奇數，求 k。

7. The graph of the parabola $y = x^2 - 4x - \frac{9}{4}$ cuts the x-axis at A and B (Figure 1). If C is the vertex of the parabola, find the area of ΔABC.

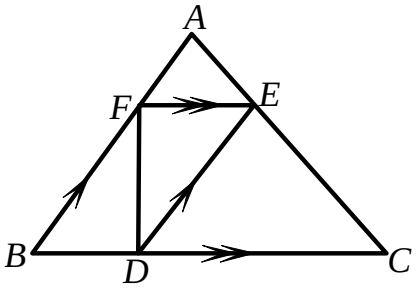
拋物線 $y = x^2 - 4x - \frac{9}{4}$ 的圖像交 x-軸於 A 及 B (圖 1)。若 C 是拋物線的頂點，求 ΔABC 的面積。



(Figure 1)(圖 1)

8. In Figure 2, $FE // BC$ and $ED // AB$. If $AF : FB = 1 : 4$, find the ratio of area of ΔEDC : area of ΔDEF.

在圖 2 中，$FE // BC$ 及 $ED // AB$。若 $AF : FB = 1 : 4$，求 ΔEDC 的面積 : ΔDEF 的面積。



(Figure 2)(圖 2)

9. In the attached multiplication (Figure 3), the letters O, L, Y, M, P, I, A and D represent different integers ranging from 1 to 9. Find the integer represented by A.

在所附乘法算式中 (圖 3)，字母 O、L、Y、M、P、I、A 及 D 代表由 1 至 9 的不同整數，求 A 所代表的整數。

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    O L Y M P I A D
×               D
  ○○○○○○○○○
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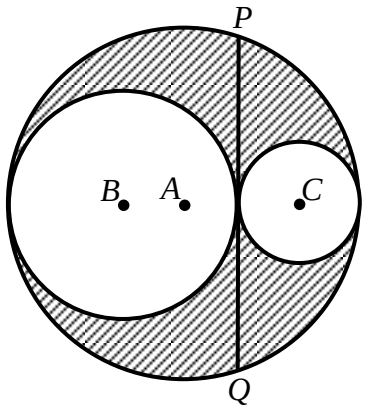
(Figure 3)(圖 3)

10. Three circles, with centres A, B and C respectively, touch one another as shown in figure 4. If A, B and C are collinear and PQ is a common tangent to the two smaller circles, where $PQ = 4$, find the area of the shaded part in terms of π.

以 A、B 及 C 為圓心的三個圓兩兩相切如圖 4。若 A、B 及 C 共線，PQ 是兩個較小圓的公切線，且 $PQ = 4$，試以 π 表陰影面積。



(Figure 4)(圖 4)